NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES
SCHOOL OF NATURAL AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: BACHELOR OF SCIENCE HONOURS IN APPLIED MATHEMATICS	
QUALIFICATION CODE: 08BSHM	**LEVEL: 8**
COURSE CODE: PDE801S	**COURSE NAME**: PARTIAL DIFFERENTIAL EQUATIONS
SESSION: JUNE 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS: 98**

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	**Prof A.S Eegunjobi**
MODERATOR:	Prof Sandile Motsa

INSTRUCTIONS

1. Answer ALL the questions.
2. Write clearly and neatly.
3. Number the answers clearly.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 2 PAGES (Including this front page)

QUESTION 1 [24 marks]

1. Form partial differential equations
   (a) by eliminating the arbitrary functions ϕ from the relation

$$xyu(x,y) = \phi(x+y+u(x,y))$$

(7)

   (b) by eliminating the arbitrary functions f and g from the relation

$$u(x,y) = f(x+\alpha y) + g(x-\alpha y)$$

(7)

   (c) by eliminating the arbitrary constants a, b and c from the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

(10)

QUESTION 2 [24 marks]

2. Solve the following first order PDE
   (a) $\frac{\partial u}{\partial x} + 3\frac{\partial u}{\partial y} = 5u + \tan(y-3x)$ (8)
   (b) $u\frac{\partial u}{\partial x} - u\frac{\partial u}{\partial y} = (x+y)^2 + z^2$ (8)
   (c) $x(y-z)\frac{\partial z}{\partial x} + y(z-x)\frac{\partial z}{\partial y} = z(x-y)$ (8)

QUESTION 3 [20 marks]

3. (a) Reduce to normal form and hence solve

$$(y-1)\frac{\partial^2 z}{\partial x^2} - (y^2-1)\frac{\partial^2 z}{\partial x \partial y} + y(y-1)\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 2ye^{2x}(1-y)^3$$

   provided $y \neq 1$ (10)

   (b) Reduce to normal form

$$z_{xx} + 2z_{xy} + 5z_{yy} + z_x - 2z_y - 3z = 0$$

(10)

QUESTION 4 [30 marks]

4. (a) Determine the displacement $y(x,t)$ for a taut string with fixed endpoints at $x = 0$ and $x = l$, initially held in position $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$ and released from rest. (15)

   (b) Find the solution of the Cauchy problem

$$u_{tt} - c^2 u_{xx} = 0, \qquad x \in \mathbb{R}, \quad t > 0, \quad u(x,0) = f(x), \quad u_t(x,0) = g(x), \quad x \in \mathbb{R}.$$

(15)

End of Exam!